How to split a Relation

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TLLM 30 March 2024

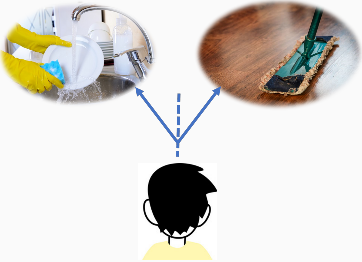
State Split vs Relation Split

State Split

It is raining **or** *it is sunny.*

State Split vs Relation Split

State Split



Relation Split

It is raining **or** *it is sunny.*

You must do the dishes **or** *you must clean the floor.*

Outline

1. The puzzle
2. Bilateral State-based Modal Logic (BSML)
3. Relation Splitting
4. Limitations
5. Generalizations
6. Conclusion

The puzzle

Free Choice Inference[1]

(1) You are allowed to watch a movie or read a book.
$\rightsquigarrow$ You are allowed to watch a movie and you are allowed to read a book.

[1] Kamp 1981, Fox 2007, Goldstein 2019, Aloni 2022

Free Choice Inference[1]

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$$\Diamond(p \vee q) \rightsquigarrow \Diamond p \wedge \Diamond q$$

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Free Choice Across Modal Force and Flavour

Free choice inferences are attested independently of modal force, flavour and the scope of disjunction (Zimmermann 2001, Aloni 2022).

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Goal: Uniform theory which predicts all observed patterns of inference.

Wide Scope disjunction

- Focus: Wide Scope disjunction of Universal modals

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- Wide Scope Deontic:

(2) (To pass the course) you must write an essay or you must solve an assignment. $\Box p \vee \Box q$

a. $\rightsquigarrow$ You are allowed to write an essay and you are allowed to solve an assignment. $\Diamond p \wedge \Diamond q$

b. $\not\rightsquigarrow$ You must write an essay and you must solve an assignment. $\Box p \wedge \Box q$

- Wide Scope Epistemic:

(3) (In this period of the year), Jialiang must be in Amsterdam or Jialiang must be in Beijing. $\Box p \vee \Box q$

a. $\rightsquigarrow$ Jialiang might be in Amsterdam and he might be in Beijing. $\Diamond p \wedge \Diamond q$

b. $\not\rightsquigarrow$ Jialiang must be in Amsterdam and he must be in Beijing. $\Box p \wedge \Box q$

The Puzzle

(4) a. Jialiang must be in Amsterdam or Jialiang must be in Beijing.
$\Box p \vee \Box q$

b. Jialiang might be in Amsterdam and Jialiang might be in Beijing.
$\Diamond p \wedge \Diamond q$

Suppose that if $\Diamond p$ then $\Diamond \neg q$ and if $\Diamond q$ then $\Diamond \neg p$
$\rightsquigarrow$ Jialiang might be not in Amsterdam and Jialiang might be not in Beijing.
$\Diamond \neg p \wedge \Diamond \neg q$

$\Box p \vee \Box q$

By Free Choice

$\Diamond p \wedge \Diamond q$

by assumption

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$$\Box p \vee \Box q$$

By Free Choice

$$\Diamond p \wedge \Diamond q$$

by assumption

$$(\Box p \vee \Box q) \wedge \Diamond \neg p \wedge \Diamond \neg q$$

by classical logic (K)

$$\bot$$

Bilateral State-based Modal Logic (BSML)

BSML and Neglect-Zero

- Aloni (2022): BSML - Bilateral State based Modal Logic

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- Free choice inferences as the result of a (cognitive) pragmatic factor called neglect-zero
- Neglect-zero: structures that vacuously satisfy a sentence due to an empty configuration are avoided

$[\blacksquare, \blacksquare, \blacksquare]$

(a) Verifier

$[\blacksquare, \square, \blacksquare]$

(b) Falsifier

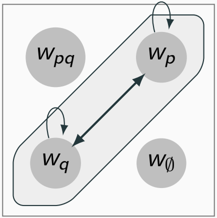
$[\ ]; [\triangle, \triangle, \triangle]; [\lozenge, \blacktriangle, \spadesuit]$

(c) Zero-models

Figure 1: Models for the sentence *Every square is black.*

BSML

- Formulas interpreted at pointed models (M, s)



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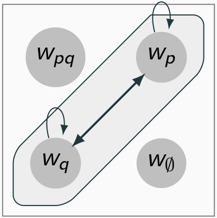

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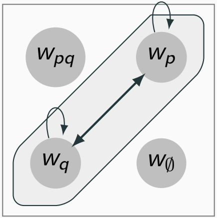

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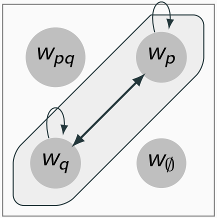

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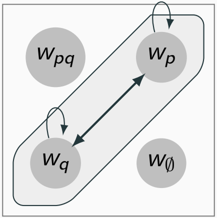

- Neglect-zero: NE atom which requires the supporting state to be non-empty
- Enrichment function $[\cdot]^+$ adding NE recursively on the complexity of the formulas

Disjunction

- Split Disjunction

$$M, s \models \phi \vee \psi \text{ } \textit{iff} \text{ } \exists t, t' : t \cup t' = s \;\&\; M, t \models \phi \;\&\; M, t' \models \psi$$

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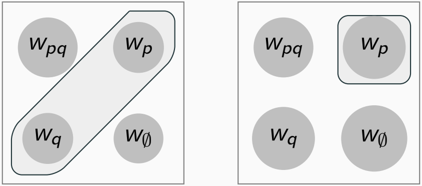


$\models p \vee q$
$\models [p \vee q]^{+}$

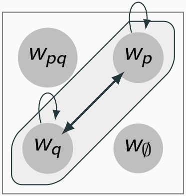
$\models p \vee q$
$\not\models [p \vee q]^{+}$

Accessibility Relation

- State-based R (epistemic).

$$R \text{ is state-based in } (M, s) \text{ iff } \forall w \in s : R[w] = s$$

Epistemic possibilities are actual possibilities.



State-based model

Accessibility Relation

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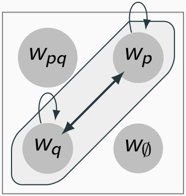
Epistemic possibilities are actual possibilities.

- Indisputable R (deontic permission).

$$R \text{ is indisputable in } (M, s) \text{ iff } \forall w, w' \in s : R[w] = R[w']$$

Full information about what is allowed and what is not allowed.



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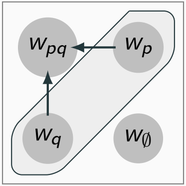

Indisputable model

Modalities in BSML

Let $R[w] = \{v \mid wRv\}$

$M, s \models \Diamond\phi$ iff $\forall w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$

$M, s \models \Box\phi$ iff $\forall w \in s : M, R[w] \models \phi$

BSML and Free Choice

- BSML predicts the attested FC inference across different cases:

$[(p \vee q)]^+$	$\models \Diamond p \wedge \Diamond q$	if R is state-based
$[\Diamond(p \vee q)]^+$	$\models \Diamond p \wedge \Diamond q$	
$[\Box(p \vee q)]^+$	$\models \Diamond p \wedge \Diamond q$	
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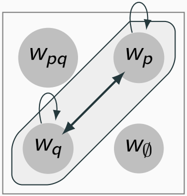

$[\Box(p \vee q)]^+ \models \Diamond p \wedge \Diamond q$

Problem[2]

$$[\Box p \vee \Box q]^+ \models \Box p \wedge \Box q \text{ if } R \text{ is indisputable}$$

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Suppose $M, s \models [\Box p \vee \Box q]^+$ then $\exists t \subseteq s$: $M, t \models \Box(p \wedge \textsc{ne})$.

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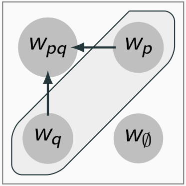
But by *indisputability:* For any $w' \in s : R[w'] = R[w]$ so $R[w'] \models p$. Thus $M, s \models \Box p$



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The puzzle for BSML

Suppose that if $\Diamond p$ then $\Diamond\neg q$ and if $\Diamond q$ then $\Diamond\neg p$
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$$[\Box p \vee \Box q]^+$$

By Free Choice

$$[\Box p \vee \Box q]^+ \wedge \Diamond p \wedge \Diamond q$$

by assumption

$$[\Box p \vee \Box q]^+ \wedge \Diamond\neg p$$

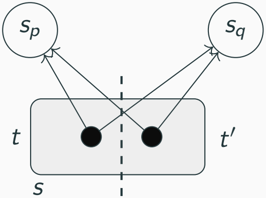
by prev. slide

$$\Box p \wedge \Diamond\neg p$$

Relation Splitting

Motivation

Disjunctions allow us to entertain *different alternatives* separately. BSML models this by *splitting the state*.



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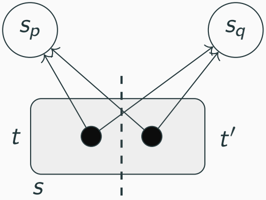

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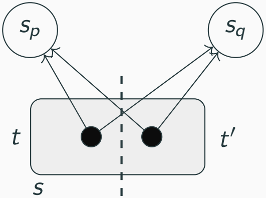

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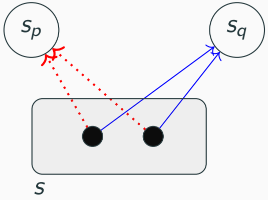


What about modal alternatives constructed from the accessibility relation?

BSML does not allow us to entertain modal alternatives separately.

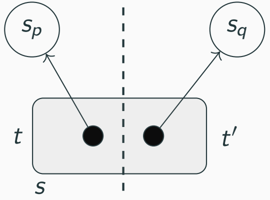
(5) You must write an essay or you must solve an assignment.

Splitting Examples

Idea: disjunction **splits the accessibility relation** and not only the state!

Relation split disjunction:

$(W, R, V), s \models \phi \vee \psi$ iff there are $t, t' \subseteq s$, where $t \cup t' = s$, and $R_t, R_{t'} \subseteq R$, such that $(W, R_t, V), t \models \phi$ and $(W, R_{t'}, V), t' \models \psi$.



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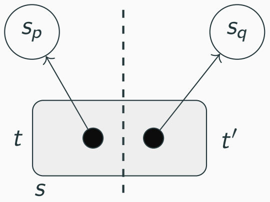

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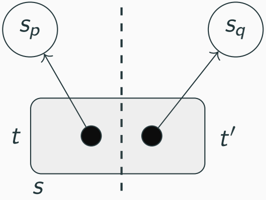

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What are the constraints on the splitting ($R_t = R_{t'} = \varnothing$)?

Constraints on Splitting

To make sure that no modal possibilities are forgotten, we impose the following constraints on possible splits:

Union: $R_t \cup R_{t'} = R$

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State-sensitiveness: For all $w \in s$, if $wR_t w'$ and not $wR_{t'} w'$, then $w \in t$, and if $wR_{t'} w'$ and not $wR_t w'$, then $w \in t'$.

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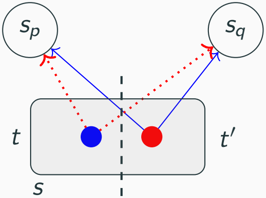
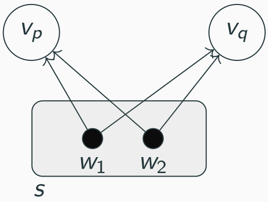
State-sensitiveness ensures that the arrows are placed in the substate where they begin.



Disallowed split

Accounting for the basic case

$[\Box p \lor \Box q]^+ \not\models \Box p \land \Box q$ even if R is indisputable:



Accounting for the basic case

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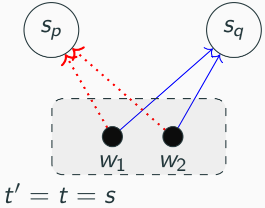
Take the following split: $s = t = t'$ and

$$R_t = \{\langle \cdot, w\rangle | w \in s_p\} \text{ and } R_{t'} = \{\langle \cdot, w\rangle | w \in s_q\}.$$

$(W, R_t, V), t \models [\Box p]^+$ and $(W, R_{t'}, V), t' \models [\Box q]^+$.

So $(W, R, V), s \models [\Box p \vee \Box q]^+$ ✓



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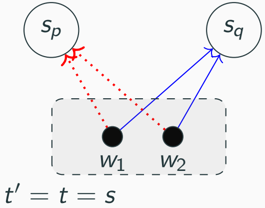

$$(W, R, V), s \models [\Box p \vee \Box q]^+$$
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$$(W, R, V), s \models \Diamond \neg p \wedge \Diamond \neg q$$
$$(W, R, V), s \not\models \Box p \wedge \Box q$$

Inferences in Relation Splitting BSML

Relation Splitting solves the main puzzle by making $(\Box p \vee \Box q)$ consistent with $\Diamond\neg p \wedge \Diamond\neg q$.

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It improves BSML since all the key inferences are preserved, but the paradoxical one is avoided:

$[(p \vee q)]^+$	$\models \Diamond p \wedge \Diamond q$	if R is state-based
$[\Diamond(p \vee q)]^+$	$\models \Diamond p \wedge \Diamond q$	
$[\Box(p \vee q)]^+$	$\models \Diamond p \wedge \Diamond q$	
$[\Diamond p \vee \Diamond q]^+$	$\models \Diamond p \wedge \Diamond q$	if R is indisputable
$[\Box p \vee \Box q]^+$	$\models \Diamond p \wedge \Diamond q$	if R is indisputable
$[\Box p \vee \Box q]^+$	$\not\models \Box p \wedge \Box q$	even if R is indisputable

BSML and Relation Splitting BSML

- $M, s \vDash_{BSML} \varphi$ implies $M, s \vDash_{RS} \varphi$
- If φ is $\Box$-free then $M, s \vDash_{RS} \varphi$ implies $M, s \vDash_{BSML} \varphi$
- If φ is $\vee$-free then $M, s \vDash_{RS} \varphi$ implies $M, s \vDash_{BSML} \varphi$

Limitations

Modal depth

Splitting of the relation works for cases like $\Box p \lor \Box q$

What about $\Box\Box p \lor \Box\Box q$?

Do higher modalities have a correspondence in natural language?

(6) ?It must be that it must be that it rains or it must be that it must be that it snows $\Box\Box p \lor \Box\Box q$

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(6) ?It must be that it must be that it rains or it must be that it must be that it snows $\Box\Box p \vee \Box\Box q$

(7) a. $\Box p :=$ In two hours it must be the case that p
b. $\Diamond p :=$ In two hours it might be the case that p
c. $\Box\Box p \vee \Box\Box q$

But the modalities are arguably not simple.

Generalizations

Arbitrary modal depth

Generalised state-sensitiveness:

If $wR_t w'$ and not $wR_{t'} w'$, then $w \in t$ or $\exists v \in t$ such that $vR_t^* w$.
and if $wR_{t'} w'$ and not $wR_t w'$, then $w \in t'$ or $\exists v \in t'$ such that $vR_{t'}^* w$

Where R_t^* is the transitive closure of R_t.

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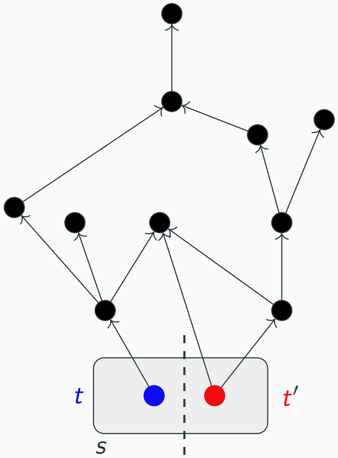
If an arrow $w_i R w_j$ is in R_t (and not in $R_{t'}$) then R_t must contain a **path** *$w_0 R_t w_1 R_t \ldots R_t w_n$ starting in t, of which this arrow is a part.*

State-sensitiveness generalised



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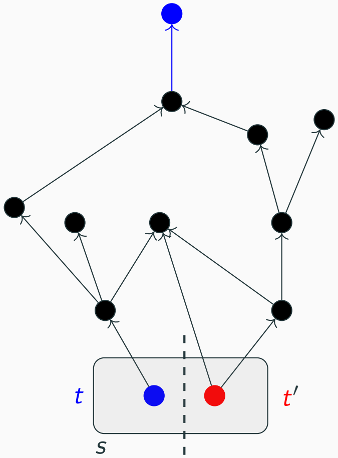

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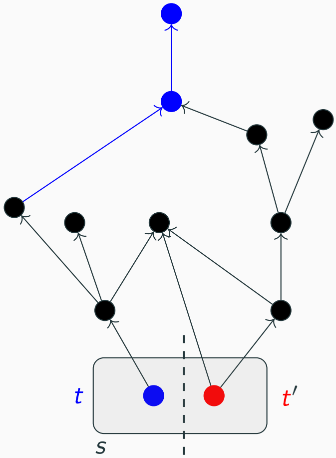

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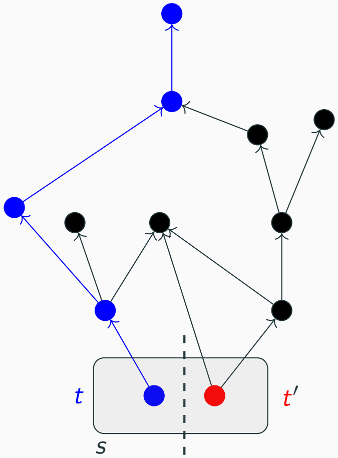

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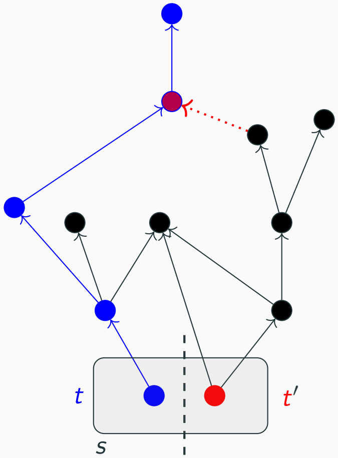

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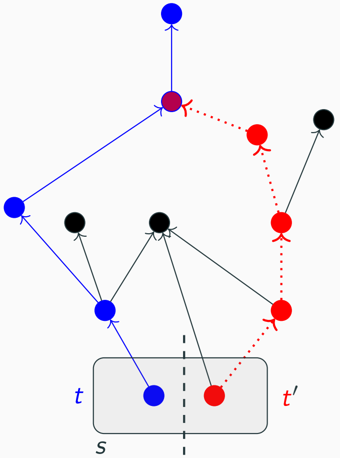

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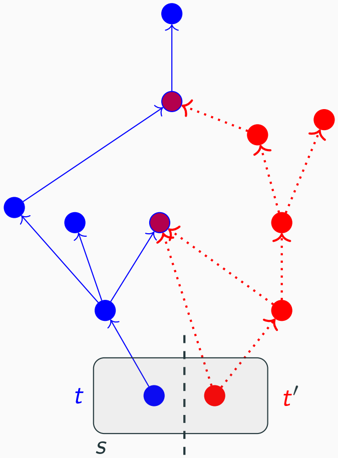
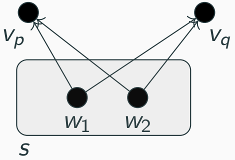


Paths

States are sets of paths starting from a given set of worlds:



In this model $s = \{(w_1, v_p), (w_1, v_q), (w_2, v_p), (w_2, v_q)\}$

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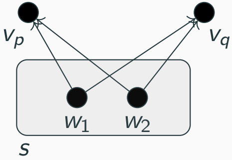


In this model $s = \{(w_1, v_p), (w_1, v_q), (w_2, v_p), (w_2, v_q)\}$

Let π denote a path and $\pi(0)$ the beginning of it e.g. $(w_1, v_p)(0) = w_1$.

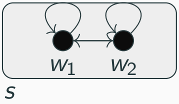
$M, s \models p$ iff for all $\pi \in s : \pi(0) \in V(p)$
$M, s \models \varphi \vee \psi$ *iff* $\exists t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$

A problematic case

(8) a. John must be in his office or he must be at home. $\Box p \vee \Box q$

b. John is in office or he must be at home. $p \vee \Box q$

c. ?John must be in his office or he is at home. $\Box p \vee q$



- Covert modal as a repair strategy?

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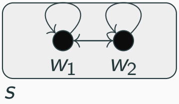

- Covert modal as a repair strategy?
- Local treatment of epistemic modals:
  $M, s \models \blacklozenge\phi$ iff $M, t \models \phi$ for some $t \subseteq s$ and $t \neq \varnothing$
  $M, s \models \blacksquare\phi$ iff $M, s \models \phi$

Conclusion

Conclusions

- Solution to the puzzle of Wide Scope Free Choice.
- Relation Splitting BSML: entertaining modal alternatives separately.
- Uniform treatment of epistemic and deontic modalities.
- Idea to explore further: states as sets of paths.

Thank you!

Epistemic Contradictions

BSML and (disjunctions of) epistemic contradictions:

- Epistemic contradictions are contradictions: $p \wedge \diamond\neg p \vDash \bot$
- But disjunctions of e.c.s need not be; in fact,

$$\diamond p, \diamond\neg p \vDash (p \wedge \diamond\neg p) \vee (\neg p \wedge \diamond p)$$

  I.e., if we are in an epistemic context where the street might be wet and it might be dry, then the following utterance is supported:

  "either the street is wet and it might be dry, or it is dry and it might be wet"[3]

- More generally, we have the following fact: For s an (epistemic) state,

$$s \vDash^{+} (p \wedge \diamond\neg p) \vee (q \wedge \diamond\neg q) \quad \textit{iff}$$
$$\forall w \in s{:}w \vDash p \vee q \textit{ and } \exists w, w' \in s{:}w \vDash p \wedge \neg q, w' \vDash \neg p \wedge q$$

Relation Splitting BSML and (disjunctions of) epistemic contradictions:

- With current version, we get the same results/predictions (also when generalising to arb. formulas, but only as long as these satisfy the conditions of our lemmas on how the different semantics relate)

[3] Notice how it sounds odd, until one reaches 'or' by which point it starts sounding tautological, as predicted by BSML: the disjunct is never supported, but the disjunction is always supported.

BSML $\subset$ RS

Corollary
$M, s \vDash_{BSML} \varphi$ implies $M, s \vDash_{RS} \varphi$

Consider a split of the relation such that $R_t = R_{t'} = R$ (*dummy split*).

Dummy splits satisfy Union (obviously) and State-sensitiveness (trivially by false the antecedent).

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Dummy splits satisfy Union (obviously) and State-sensitiveness (trivially by false the antecedent).

Observe that BSML is RS where only dummy splits are allowed! The result follows.

When RS = BSML?

Consider a $\Box$-free formula φ[4].

Lemma
$R' \supseteq R$ *then* $M, s, R \vDash_{RS} \varphi$ *implies* $M, s, R' \vDash_{RS} \varphi$

Proof by induction: if $\varphi = \psi \vee \chi$ then $R'_\psi := R_\psi \cup (R' \setminus R)$.

[4] formulas in negation normal form without any occurrence of $\Box$.

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The minimal model

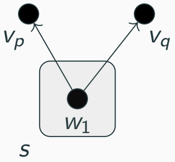

Figure 2: The minimal model